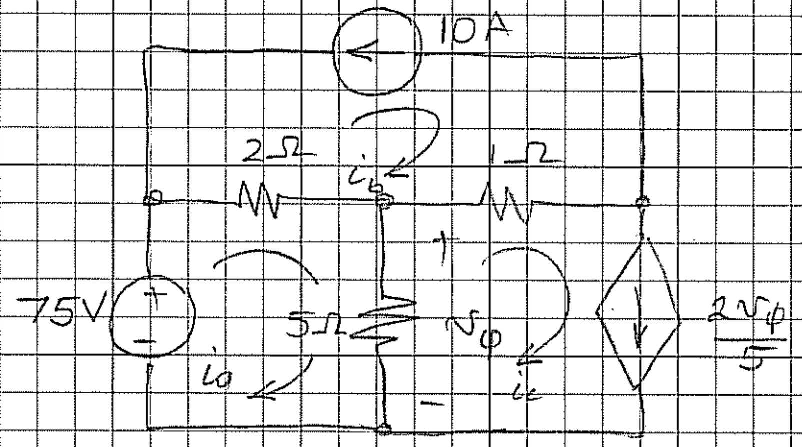
Assessment Problem 4.10:

Use the mesh-current method to find the mesh current i_a in the following circuit



$$i_b = -10\,A \qquad (1)$$

$$i_c = \frac{2v_\phi}{5} \quad \text{but } v_\phi = 5(i_a - i_c) \Rightarrow$$

$$\frac{5i_c}{2} = 5(i_a - i_c) \Rightarrow 2i_a - 3i_c = 0 \qquad (2)$$

KVL for mesh a:

$$7i_a - 2i_b - 5i_c = 75 \qquad (3)$$

Replace i_b in 3 and rearrange:

$$7i_a - 5i_c = 55 \qquad (3a)$$

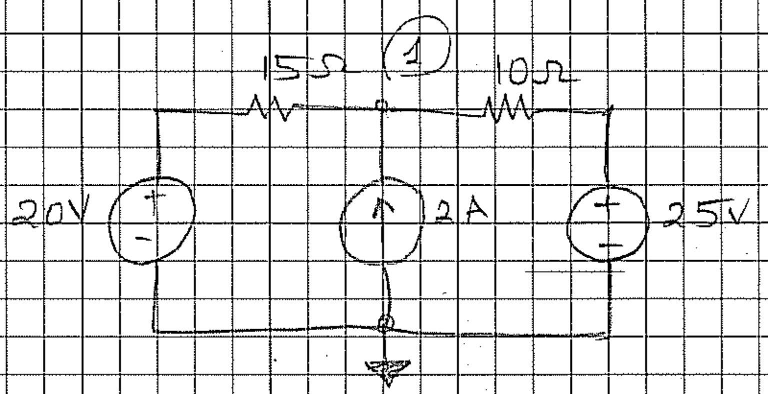
Solve (2) and (3a):

$$i_a = 15\,A \quad i_c = 10\,A$$

4.8 The Node Voltage Method Versus The Mesh-Current Method.

Assessment Problems:

4.13 Find the Power delivered by the 2A current Source:



The mesh-current method involves a super-mesh to avoid the 2A current source. In addition we need the equation of the current source. From the mesh we still need to determine the voltage of the current source.

However, the node voltage method involves only one equation for node 1, from which we directly can deduce the power delivered by the source:

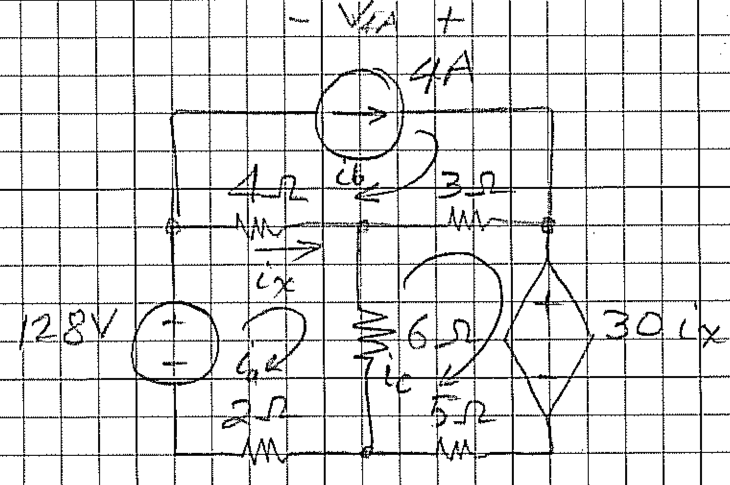
So:

KCL at 1: $\frac{v_1 - 20}{15} - 2 + \frac{v_1 - 25}{10} = 0$

$$v_1\left(\frac{1}{15} + \frac{1}{10}\right) = 5.833 \Rightarrow$$

$v_1 = 35V.$ $\qquad P_{2A} = 2 \times 35 = 70W$

4.14 Find the power delivered by the 4A current source:



The node-voltage method would involve 3 equations in 3 unknowns.

The Mesh-current method also requires 3 equations in 3 unknowns. But one of the mesh current currents would be equal to 4A.

KCL in Mesh a:

$$1(i_a - 4) + 6(i_a - i_c) + 2i_a = 128 \qquad (1)$$

$$i_b = 4 \qquad (2)$$

KCL in Mesh c:

$$3(i_c - 4) + 30\underbrace{i_x}_{i_a-4} + 5i_c + 6(i_c - i_a) = 0 \qquad (3)$$

with $i_x = i_a - 4$

Substitute $i_x = i_a - 4$ in (3) and rearrange along with (1):

$$24i_a + 14i_c = 132 \qquad (3a)$$

$$12i_a - 6i_c = 144 \qquad (1a)$$

$$i_a = 9A, \quad i_c = -6A$$

The voltage rise across the 4A source is the negative of the voltage drop across the 4Ω resistor plus the voltage drop across 3Ω resistor:

$$V_{4\Omega} = 4(i_a - 4) = 4(9-4) = 20V$$

$$V_{3\Omega} = 3(i_c - 4) = 3(-6-4) = -30V$$

$$V_{4A} = -(V_{4\Omega} + V_{3\Omega}) = -(20-30) = 10V$$

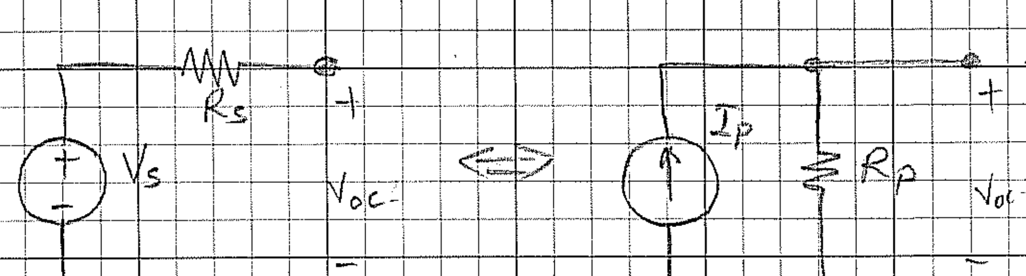
$$P_{4A} = 4V_{4A} = 4 \times 10 = 40W$$

The 4A source is delivering 40W to the circuit.

4.9 Source Transformations

This is another tool to help in circuit simplification. to add to series-parallel reductions and Δ-Y transformations

Here a voltage source with a series resistance is to be made equivalent to a current source with a parallel resistance.



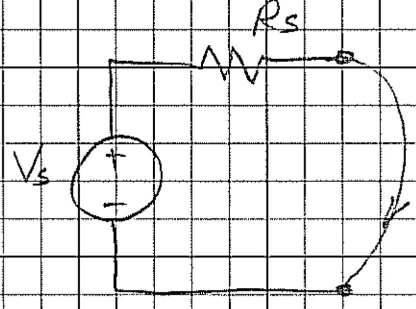
For the two sources to be equivalent they should have the same short circuit current when they are shorted and the same voltage when they are open.

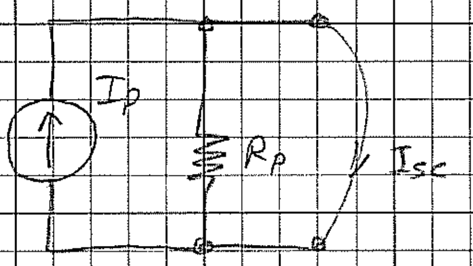
The open circuit condition is shown above:

$V_{oc} = V_s$ from the voltage source circuit. (1)

$V_{oc} = I_p R_p$ from the current source circuit (2)

Under short-circuit conditions, the situation is as follows:





From the voltage source circuit:

$$I_{sc} = \frac{V_s}{R_s} \quad (3)$$

From the current source circuit, noting that no current flows in R_p because it is shorted:

$$I_{sc} = I_p \quad (4)$$

From the above 4 equation we deduce that:

$V_s = I_p R_p$ and $I_p = \frac{V_s}{R_s}$

and $R_p = R_s$!

At any other load Resistance R_L the voltage and currents are:

* From the voltage source:

$$V_L = V_s \frac{R_L}{R_L + R_s} \quad \text{and} \quad I_L = \frac{V_s}{R_L + R_s}$$

* From the current source and using $R_p = R_s$ and $I_p = V_s / R_s$:

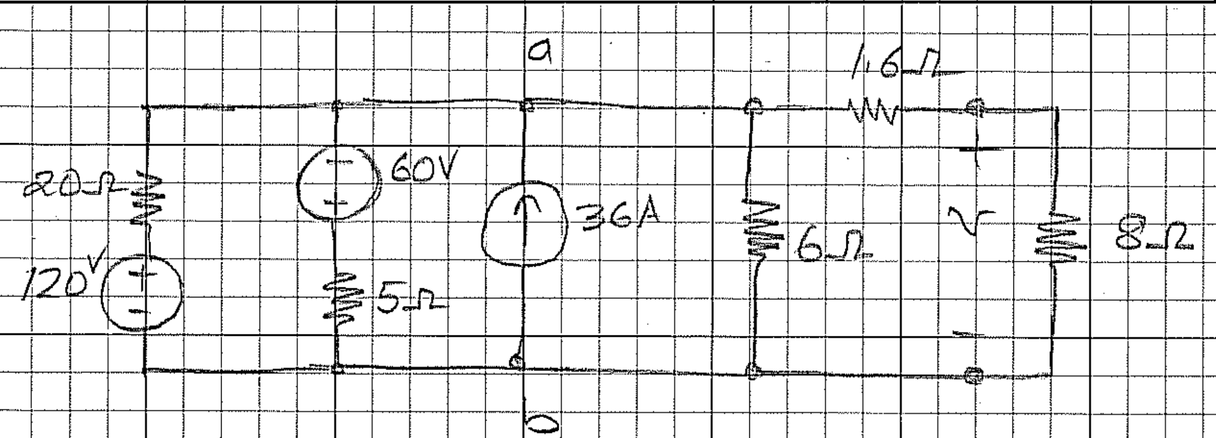
$$V_L = I_p \frac{R_L R_p}{R_L + R_p} = V_s \frac{R_L}{R_L + R_s}$$

$$I_L = I_p \frac{R_p}{R_L + R_p} = \frac{V_s}{R_L + R_s}$$

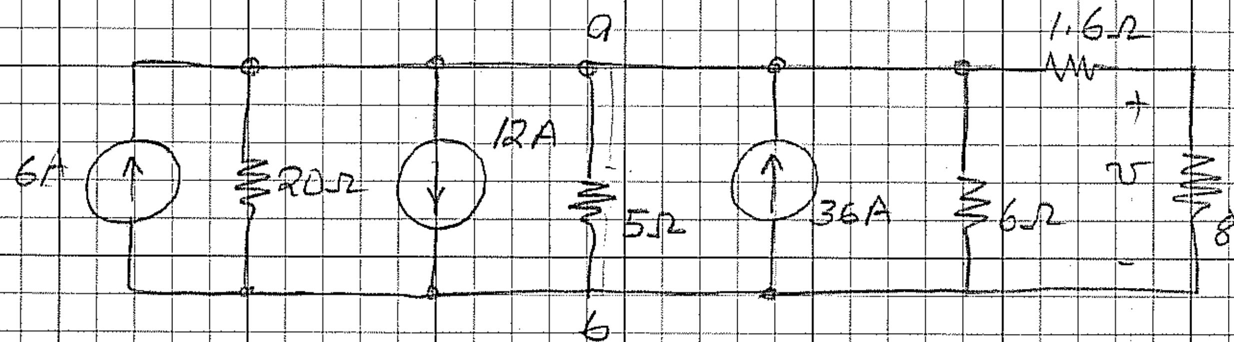
Assessment Problem 4.15:

a) Use Source transformations to find the voltage v in the circuit shown below.

b) What is the power delivered by the 120V source to the circuit.



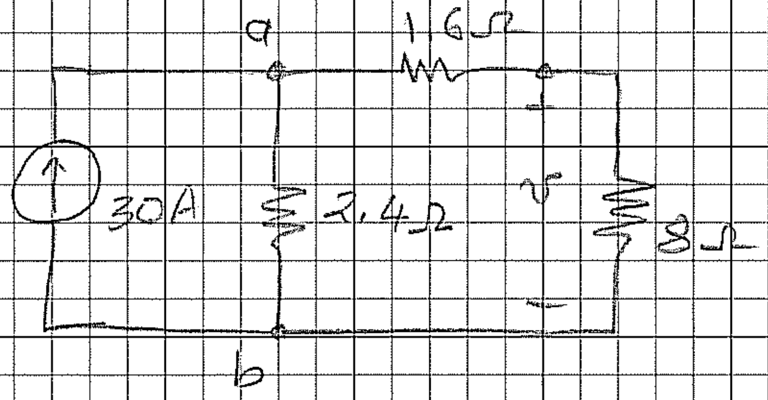
Transform the two voltage sources into current sources to obtain the following circuit:



The 3 current source appear in parrallel between nodes a and b. They can be replaced by one source of 30A (6−12+36) going up. The three resistances 20, 5, and 6 Ω appear in parrallel. Their equivalent resistance is:

$$R_{eq} = \left(\frac{1}{20} + \frac{1}{5} + \frac{1}{6}\right)^{-1} = 2.4\,\Omega$$

So the circuit becomes:



By current division, the current in the 8Ω is:

$$I_{8\Omega} = 30 \times \frac{2.4}{(1.6+8)+2.4} = 6\,A$$

So $v = 6 \times 8 = 48\,V$.

The current out of the 120V source is:

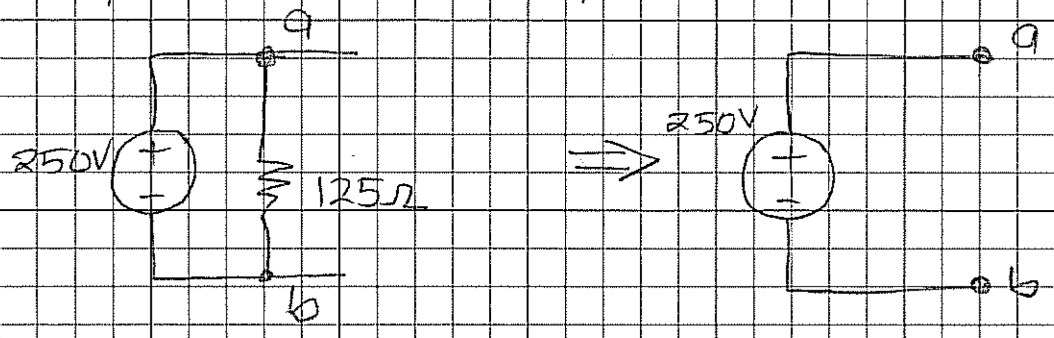
$$I_{120V} = \frac{120 - V_{ab}}{20}$$

But $V_{ab} = 6 \times (1.6+8) = 57.6\,V$

So $I_{120V} = 3.12\,A$

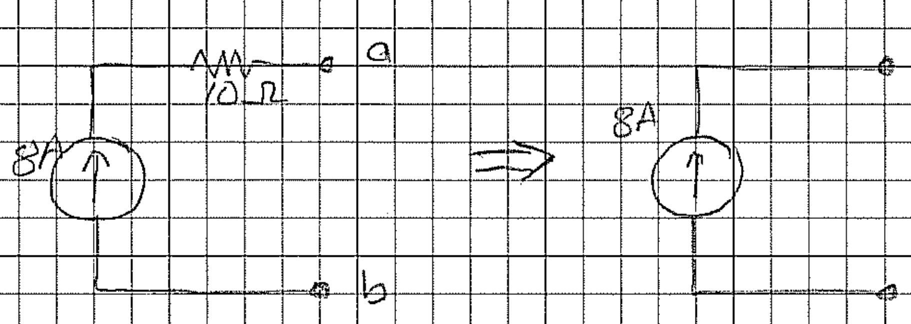
$$P_{120V} = 120 \times 3.12 = 374.4\,W$$

Special source simplifications:



because the voltage between points a and b is 250V whether the 125Ω resistor is there or not.

Furthermore

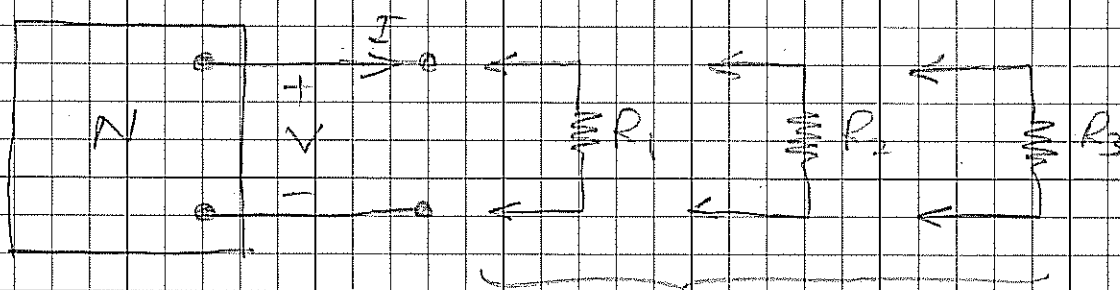


Because the current out of a (and into b) is 8A irrespective of the presence of the 10Ω resistor.

Note that this simplification is valid for calculating the voltage Vab and the current out of "a". The power supplied by the sources would be different in the two cases.

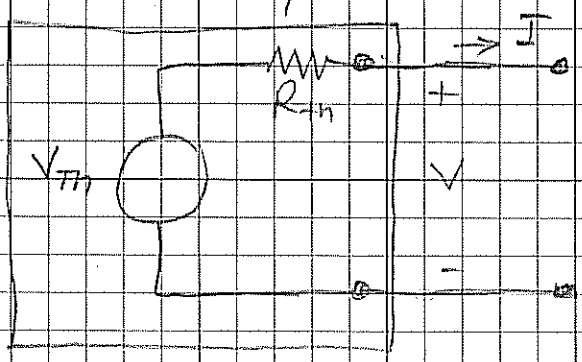
4.10 Thevenin and Norton Equivalents.

Some times in circuit analysis we want to concentrate on the voltage and current at a specific pair of terminals as different loads are connected. For example what happens inside Network N is not of direct interest if we want to calculate the power dissipated in the different load:



Network
different loads.

The Network can best be described by its Thevenin equivalent as shown:



The Thevenin equivalent should describe the network behaviour at open circuit and Short circuit. The open circuit of the Circuit must be equal to the open circuit voltage of the Thevenin equivalent;

$V_{oc} = V_{Th}$ (1)

And the short circuit current of the network should be:

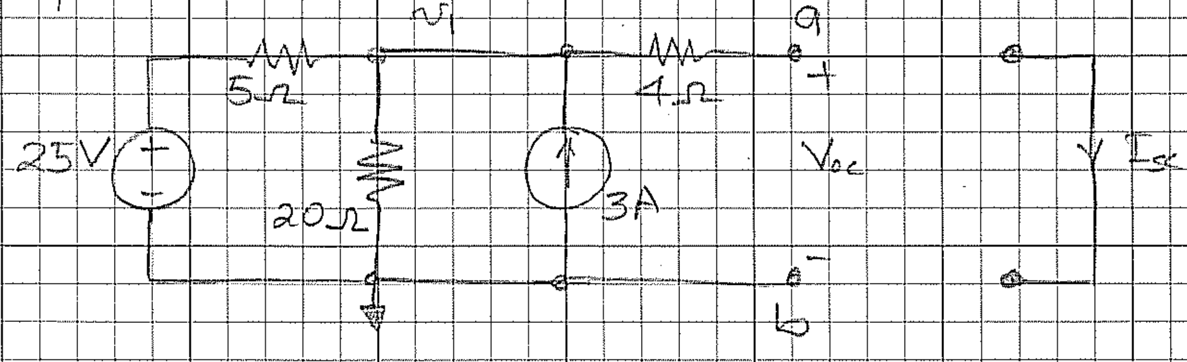
$$I_{sc} = \frac{V_{Th}}{R_{Th}} \qquad (2)$$

If V_{oc} and I_{sc} are determined by measurement or circuit analysis then the Thevenin equivalent parameters V_{Th} and R_T can be determined from (1) and (2) as:

$$V_{Th} = V_{oc} \quad \text{and} \quad R_{Th} = \frac{V_{oc}}{I_{sc}}$$

Example of Finding a Thevenin Equivalent:

For the network shown below find the Thevenin equivalent between terminals a and b:



The voltage V_{oc} is equal to the voltage v_1, which can be found using the NVM:

$$\frac{v_1 - 25}{5} + \frac{v_1}{20} - 3 = 0 \Rightarrow v_1 = \frac{8}{0.25} = 32V.$$

Under Short Circuit Conditions the KCL at node 1 is:

$$\frac{v_1 - 25}{5} + \frac{v_1}{20} - 3 + \frac{v_1}{4} = 0$$

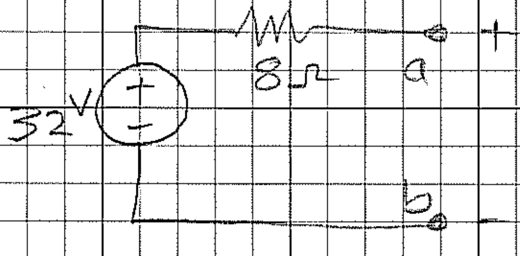
The current $\frac{v_1}{4}$ is the current that flows out of a when the network is shorted between a and b. By solving the above equation:

$$v_1 = \frac{8}{0.5} = 16V$$

$$I_{sc} = \frac{16}{4} = 4A$$

So $V_{Th} = V_{oc} = 32V$ and $R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{32}{4} = 8\Omega$

So the Thevenin Equivalent is:



The Norton equivalent has a current source in parallel with a resistor. It can be found by circuit analysis under open and short-circuit conditions:

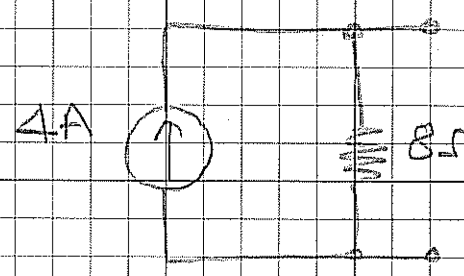
$$R_N = \frac{V_{oc}}{I_{sc}} \quad \text{and} \quad I_N = I_{sc}$$

This is illustrated in the textbook. The Noton equivalent may also be found by source transformation on the Thevenin equivalent:

$$R_N = R_{Th} = 8\,\Omega$$

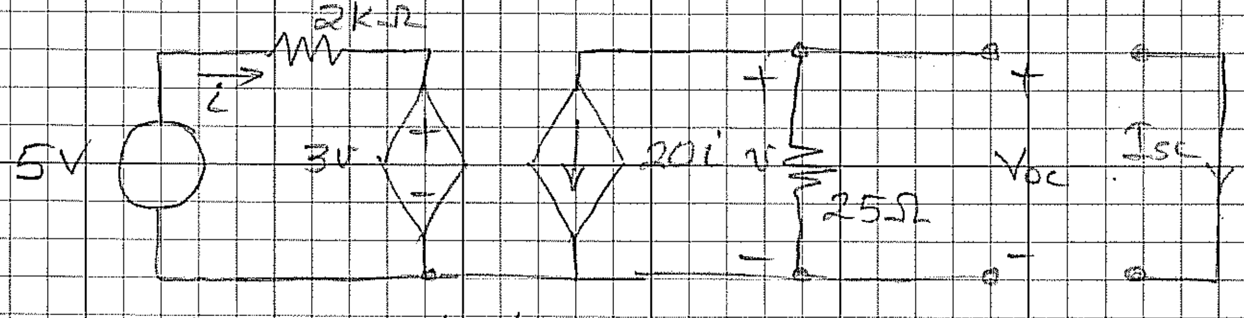
$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{32}{8} = 4\text{ A}$$

So the Noton equivalent is:



Example 4.10

Find the Thevenin equivalent of the following amplifier circuit model:



For the output loop:

$$V_{oc} = -20i \times 25 = -500i \qquad (1)$$

The current is given by:

$$i = \frac{5 - 3v}{2}$$

But in this case $v = V_{oc}$ so $i = \frac{5-3V_{oc}}{2}$

Replace in (1) to obtain:

$$V_{oc} = -500 \times \frac{(5-3V_{oc})}{2000} \Rightarrow$$

$$V_{oc} = -1.25 + 0.75\,V_{oc}$$

$$V_{oc} = \frac{-1.25}{0.25} = -5\text{ V}$$

Under Short Circuit Conditions:

$$I_{sc} = -20i \quad (\text{with } v = 0!)$$

From the input loop:

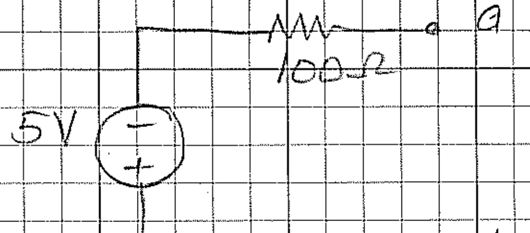
$$i = \frac{5-3v}{2} = \frac{5}{2000} = 2.5\times10^{-3} = 2.5\text{ mA}$$

So $\quad I_{sc} = -20\times2.5 = -50\text{ mA}$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{-5}{-50\times10^{-3}} = 100\ \Omega$$

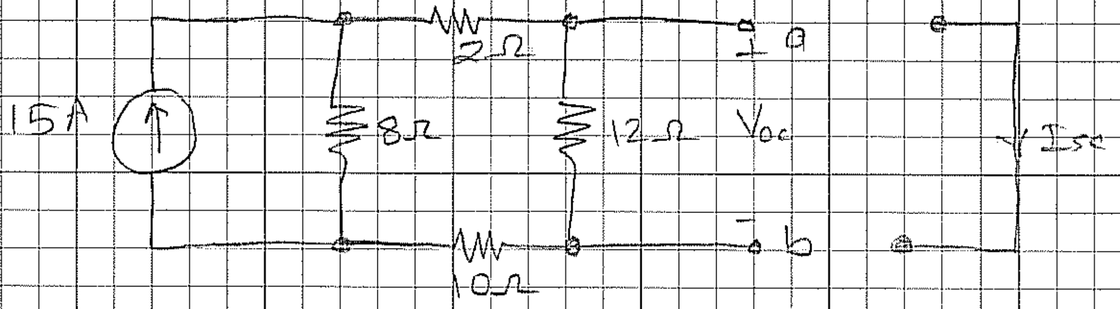
$$V_{Th} = -5\text{V}$$

The Thevenin Equivalent is:



Assessment Problem 4.17

Find the Norton Equivalent with respect to terminals "a" and "b" in the following circuit:



$I_N = I_{sc}$ and $R_N = \dfrac{V_{oc}}{I_{sc}}$

Under SC the 12Ω resistor is shorted! The short circuit current is the current in 2Ω (and the 10Ω) resistors. This current can be found by the current division rule:

$$I_{sc} = 15 \times \frac{8}{8+(2+10)} = 6A \quad \left(= 15 \times \frac{4.8}{12}\right)$$

$I \times R_{eq}/R_j$

The open circuit Voltage is the voltage across the 12Ω resistor, which can be deduced from the current that flows in 12Ω resistor. This current can also be found using Current division:

$$I_{12\Omega} = 15 \times \frac{8}{8+(2+12+10)} = 3.75\ A$$

So the open circuit voltage is :

$$V_{oc} = 12 \times 3.75 = 45\text{ V}$$

$$R_N = \frac{V_{oc}}{I_{sc}} = \frac{45}{6} = 7.5\,\Omega$$

So the Norton Equivalent is :